

The Standard Model

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- Gauge Invariance: QED, QCD
- Electroweak Unification: $SU(2)_L \otimes U(1)_Y$
- Symmetry Breaking: Higgs Mechanism
- Electroweak Phenomenology
- Flavour Dynamics

Quarks



up



down



charm



strange



top



beauty

Leptons



electron



neutrino e



muon



neutrino μ



tau



neutrino τ

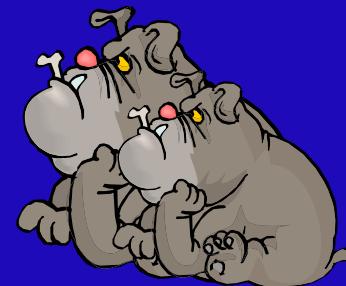
Bosons



photon



gluon



Z⁰ W±



Higgs

FREE Dirac fermion:

$$\mathcal{L} = \bar{\psi} (i \gamma^\mu \partial_\mu - m) \psi$$

Phase Invariance: $\psi \rightarrow \psi' = e^{iQ\theta} \psi$; $\bar{\psi} \rightarrow \bar{\psi}' = e^{-iQ\theta} \bar{\psi}$

Absolute phases are not observable in Quantum Mechanics

GAUGE PRINCIPLE: $\theta = \theta(x)$

Phase Invariance should hold LOCALLY

BUT

$$\partial_\mu \psi \rightarrow e^{iQ\theta} (\partial_\mu + i Q \partial_\mu \theta) \psi$$

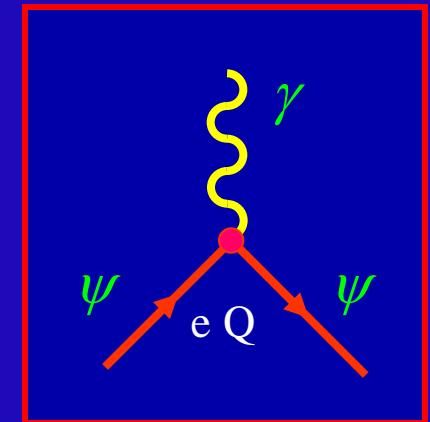
SOLUTION: Covariant Derivative

$$D_\mu \psi \equiv (\partial_\mu + i e Q A_\mu) \psi \rightarrow e^{iQ\theta} D_\mu \psi$$

One needs a spin-1 field A_μ satisfying $A_\mu \rightarrow A_\mu - \frac{1}{e} \partial_\mu \theta$

QUANTUM ELECTRODYNAMICS

$$\begin{aligned}\mathcal{L} &= \bar{\psi} (i\gamma^\mu D_\mu - m) \psi \\ &= \bar{\psi} (i\gamma^\mu \partial_\mu - m) \psi - e Q A_\mu (\bar{\psi} \gamma^\mu \psi)\end{aligned}$$



Kinetic term:

$$\mathcal{L}_K = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

$$\partial_\mu F^{\mu\nu} = e Q (\bar{\psi} \gamma^\nu \psi)$$

Maxwell

Mass term:

[exp: $m_\gamma < 1 \cdot 10^{-18}$ eV]

$$\mathcal{L}_M = \frac{1}{2} m_\gamma^2 A^\mu A_\mu$$

Not Gauge Invariant



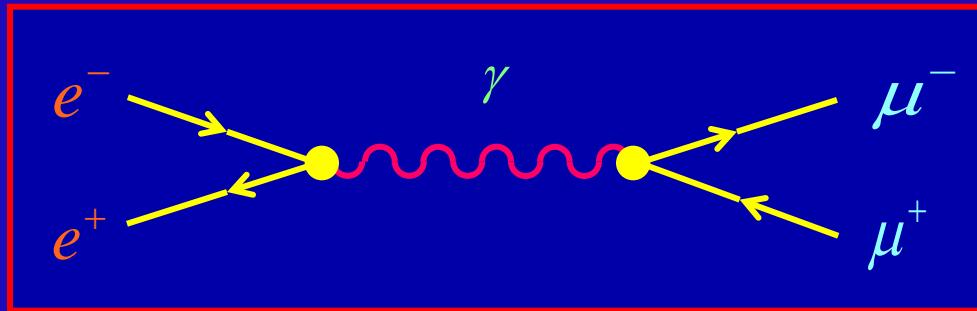
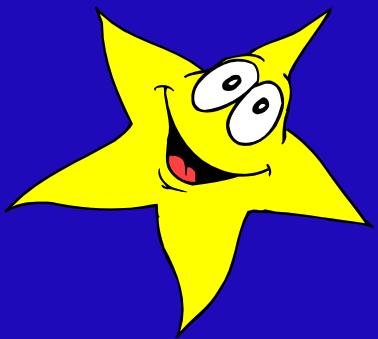
$$m_\gamma = 0$$

Gauge Symmetry



QED Dynamics

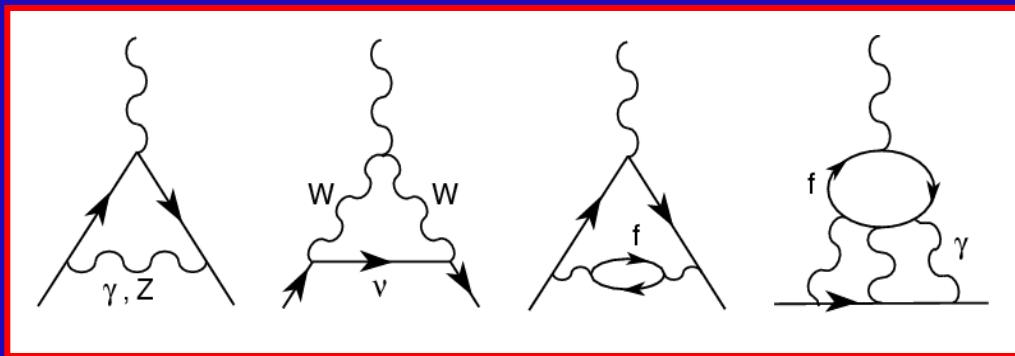
Successful Theory



Anomalous Magnetic Moment

$$\mu_l \equiv g_l \frac{e}{2m_l}$$

$$a_l \equiv \frac{1}{2} (g_l - 2)$$



$$a_e = (1\ 159\ 652\ 180.73 \pm 0.28) \times 10^{-12} \quad \rightarrow \quad \alpha^{-1} = 137.035\ 999\ 084 \pm 0.000\ 000\ 051$$

$$\rightarrow \quad a_\mu^{\text{th}} = (11\ 659\ 184 \pm 5) \times 10^{-10}$$

[Exp: $(11\ 659\ 208.9 \pm 6.3) \times 10^{-10}$]

QUANTUM CHROMODYNAMICS

$$\mathbf{q} \equiv \begin{pmatrix} q \\ q \\ q \end{pmatrix}$$

FREE QUARKS: $\mathcal{L} = \bar{\mathbf{q}} [i \gamma^\mu \partial_\mu - m] \mathbf{q}$ $N_c = 3$

SU(3) Colour Symmetry: $\mathbf{q} \rightarrow \mathbf{U} \mathbf{q}$; $\bar{\mathbf{q}} \rightarrow \bar{\mathbf{q}} \mathbf{U}^\dagger$

$$\mathbf{U} \mathbf{U}^\dagger = \mathbf{U}^\dagger \mathbf{U} = \mathbf{1} \quad ; \quad \det \mathbf{U} = 1 \quad ; \quad \mathbf{U} = \exp \left\{ i \frac{\lambda^a}{2} \theta_a \right\}$$

Gauge Principle: Local Symmetry $\theta_a = \theta_a(x)$

$$\mathbf{D}^\mu \mathbf{q} \equiv (\mathbf{I}_3 \partial^\mu + i g_s \mathbf{G}^\mu) \mathbf{q} \rightarrow \mathbf{U} \mathbf{D}^\mu \mathbf{q}$$

$$\mathbf{D}^\mu \rightarrow \mathbf{U} \mathbf{D}^\mu \mathbf{U}^\dagger \quad ; \quad \mathbf{G}^\mu \rightarrow \mathbf{U} \mathbf{G}^\mu \mathbf{U}^\dagger + \frac{i}{g_s} (\partial^\mu \mathbf{U}) \mathbf{U}^\dagger$$

$$[\mathbf{G}^\mu]_{\alpha\beta} \equiv \frac{1}{2} (\lambda^a)_{\alpha\beta} G_a^\mu(x)$$

8 Gluon Fields

Infinitesimal SU(3) Transformation:

$$\delta q^\alpha = i \delta\theta_a \left(\frac{\lambda^a}{2} \right)_{\alpha\beta} q^\beta \quad ; \quad \delta G_a^\mu = -\frac{1}{g_s} \partial^\mu (\delta\theta_a) - f^{abc} \delta\theta_b G_c^\mu$$

Non Abelian Group:

$$\left[\frac{\lambda^a}{2}, \frac{\lambda^b}{2} \right] = i f^{abc} \frac{\lambda^c}{2}$$

- δG_a^μ depends on G_a^μ
- Universal g_s
- Same colour charge for all quarks

Kinetic Term:

$$\mathbf{G}^{\mu\nu} \equiv -\frac{i}{g_s} [\mathbf{D}^\mu, \mathbf{D}^\nu] = \partial^\mu \mathbf{G}^\nu - \partial^\nu \mathbf{G}^\mu + i g_s [\mathbf{G}^\mu, \mathbf{G}^\nu] \rightarrow \mathbf{U} \mathbf{G}^{\mu\nu} \mathbf{U}^\dagger$$

$$\mathbf{G}^{\mu\nu} \equiv \frac{\lambda^a}{2} G_a^{\mu\nu} ; \quad G_a^{\mu\nu} = \partial^\mu G_a^\nu - \partial^\nu G_a^\mu - g_s f^{abc} G_b^\mu G_c^\nu$$

$$\mathcal{L}_K = -\frac{1}{2} \text{Tr} (\mathbf{G}^{\mu\nu} \mathbf{G}_{\mu\nu}) = -\frac{1}{4} G_a^{\mu\nu} G_{\mu\nu}^a$$

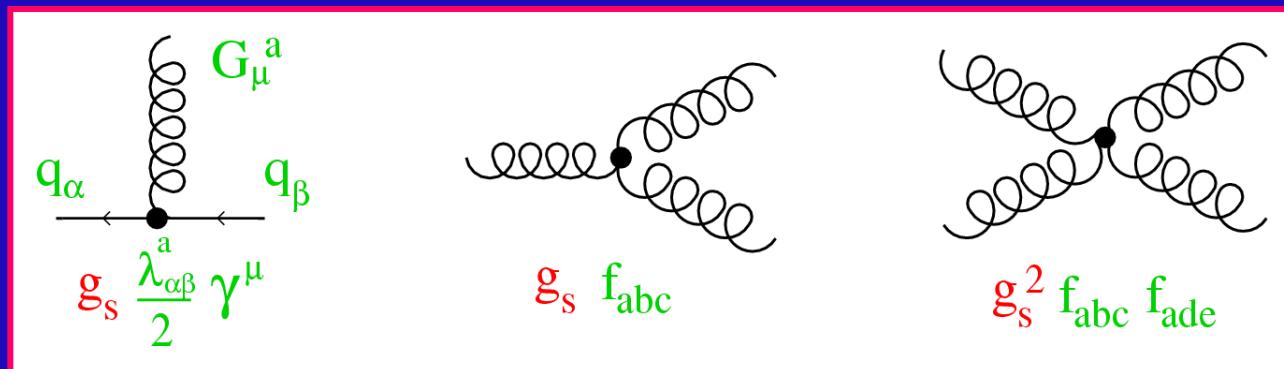
Mass Term:

$$\mathcal{L}_M = \frac{1}{2} m_G^2 G_a^\mu G_\mu^a$$

Not Gauge Invariant $\longrightarrow m_G = 0$

Massless Gluons

$$\begin{aligned}
\mathcal{L}_{\text{QCD}} &= -\frac{1}{2} \text{Tr} (\mathbf{G}^{\mu\nu} \mathbf{G}_{\mu\nu}) + \bar{\mathbf{q}} [i \gamma^\mu \mathbf{D}_\mu - m_q] \mathbf{q} \\
&= -\frac{1}{4} \left(\partial^\mu G_a^\nu - \partial^\nu G_a^\mu \right) \left(\partial_\mu G_\nu^a - \partial_\nu G_\mu^a \right) + \sum_q \bar{q}_\alpha [i \gamma^\mu \partial_\mu - m_q] q_\alpha \\
&- \frac{1}{2} \sum_q g_s [\bar{q}_\alpha (\lambda^a)_{\alpha\beta} \gamma^\mu q_\beta] G_\mu^a \\
&+ \frac{1}{2} g_s f_{abc} (\partial_\mu G_\nu^a - \partial_\nu G_\mu^a) G_b^\mu G_c^\nu - \frac{1}{4} g_s^2 f_{abc} f_{ade} G_b^\mu G_c^\nu G_\mu^d G_\nu^e
\end{aligned}$$



- **Gluon Self – interactions** $\mathbf{G}^3, \mathbf{G}^4$
- **Universal Coupling** g_s **(No Colour Charges)**

EXPERIMENTAL FACTS

Three Families

$$\begin{bmatrix} \nu_e & u \\ e^- & d' \end{bmatrix} , \quad \begin{bmatrix} \nu_\mu & c \\ \mu^- & s' \end{bmatrix} , \quad \begin{bmatrix} \nu_\tau & t \\ \tau^- & b' \end{bmatrix}$$

Family
Structure

$$\begin{bmatrix} \nu_l & q_u \\ l^- & q_d \end{bmatrix} \equiv \left\{ \begin{pmatrix} \nu_l \\ l^- \end{pmatrix}_L, (\nu_l)_R, l_R^- \right\} ; \quad \left\{ \begin{pmatrix} q_u \\ q_d \end{pmatrix}_L, (q_u)_R, (q_d)_R \right\}$$

Charged Currents

$$W^\pm \begin{cases} \text{Left-handed Fermions only} \\ \text{Flavour Changing: } \nu_l \Leftrightarrow l, \quad q_u \Leftrightarrow q_d \end{cases}$$

Neutral currents

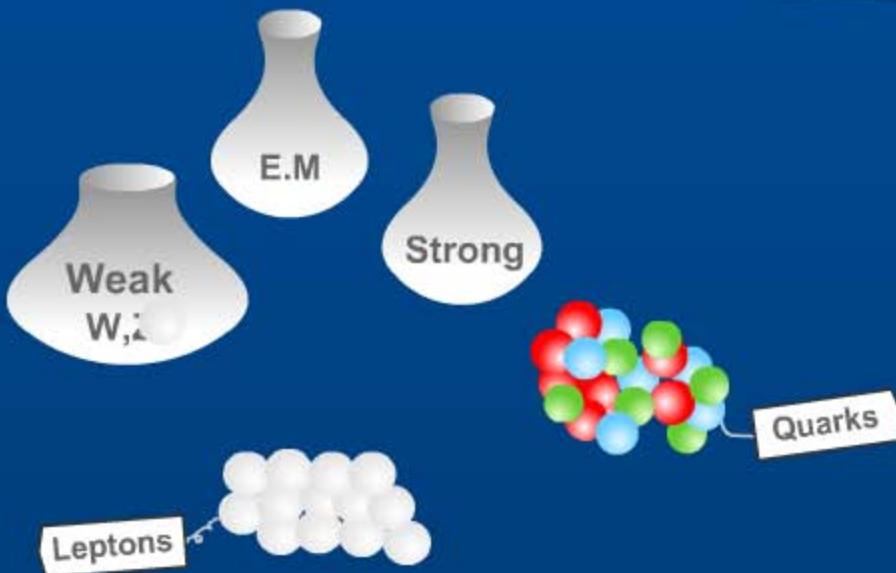
γ, Z Flavour Conserving

Universality

(Family – Independent Couplings)

$$(\nu_l)_R \quad ?$$

standard model



mfp

$$\text{SU}(2)_L \otimes \text{U}(1)_Y$$

GAUGE THEORY

Fields	$\psi_1(x)$	$\psi_2(x)$	$\psi_3(x)$
Quarks	$\begin{pmatrix} q_u \\ q_d \end{pmatrix}_L$	$(q_u)_R$	$(q_d)_R$
Leptons	$\begin{pmatrix} \nu_l \\ l^- \end{pmatrix}_L$	$(\nu_l)_R$	$(l^-)_R$

Free Lagrangian for Massless Fermions:

$$\mathcal{L}_0 = \sum_j i \bar{\psi}_j \gamma^\mu \partial_\mu \psi_j$$

$$\text{SU}(2)_L \otimes \text{U}(1)_Y$$

Flavour Symmetry:

$$U_L \equiv \exp \left\{ i \frac{\vec{\sigma}}{2} \vec{\alpha} \right\}$$

$$\psi_1 \rightarrow e^{i y_1 \beta} U_L \psi_1 \quad ; \quad \psi_2 \rightarrow e^{i y_2 \beta} \psi_2 \quad ; \quad \psi_3 \rightarrow e^{i y_3 \beta} \psi_3$$

$$\bar{\psi}_1 \rightarrow \bar{\psi}_1 U_L^\dagger e^{-i y_1 \beta} \quad ; \quad \bar{\psi}_2 \rightarrow \bar{\psi}_2 e^{-i y_2 \beta} \quad ; \quad \bar{\psi}_3 \rightarrow \bar{\psi}_3 e^{-i y_3 \beta}$$

4 Massless Gauge Bosons

$$W_\mu^\pm, W_\mu^3, B_\mu^0$$

Gauge Principle: $\vec{\alpha} = \vec{\alpha}(x)$, $\beta = \beta(x)$

$$\mathbf{D}_\mu \psi_1 \equiv \left[\partial_\mu + i g \mathbf{W}_\mu(x) + i g' y_1 B_\mu(x) \right] \psi_1 \rightarrow e^{i y_1 \beta(x)} \mathbf{U}_L(x) \mathbf{D}_\mu \psi_1$$

$$\mathbf{D}_\mu \psi_k \equiv \left[\partial_\mu + i g' y_k B_\mu(x) \right] \psi_k \rightarrow e^{i y_k \beta(x)} \mathbf{D}_\mu \psi_k \quad (k = 2, 3)$$

$$B_\mu(x) \rightarrow B_\mu(x) - \frac{1}{g'} \partial_\mu \beta(x)$$

$$\mathbf{W}_\mu(x) \rightarrow \mathbf{U}_L(x) \mathbf{W}_\mu(x) \mathbf{U}_L^\dagger(x) + \frac{i}{g} \partial_\mu \mathbf{U}_L(x) \mathbf{U}_L^\dagger(x)$$

$$\mathbf{U}(x) \equiv \exp \left\{ i \frac{\vec{\sigma}}{2} \vec{\alpha}(x) \right\} ; \quad \mathbf{W}_\mu(x) \equiv \frac{\vec{\sigma}}{2} \vec{W}_\mu(x) ; \quad \delta W_\mu^i = -\frac{1}{g} \partial_\mu (\delta \alpha^i) - \epsilon^{ijk} \delta \alpha^j W_\mu^k$$

4 Massless Gauge Bosons

$$W_\mu^\pm , W_\mu^3 , B_\mu^0$$

CHARGED CURRENTS

$$\sum_j i \bar{\psi}_j \gamma^\mu D_\mu \psi_j \quad \rightarrow \quad -g \bar{\psi}_1 \gamma^\mu W_\mu \psi_1 - g' B_\mu \sum_j y_j \bar{\psi}_j \gamma^\mu \psi_j$$

$$W_\mu \equiv \frac{\vec{\sigma}}{2} \cdot \vec{W}_\mu = \frac{1}{2} \begin{pmatrix} W_\mu^3 & \sqrt{2} W_\mu^\dagger \\ \sqrt{2} W_\mu & -W_\mu^3 \end{pmatrix} ; \quad W_\mu \equiv (W_\mu^1 + i W_\mu^2) / \sqrt{2}$$

$$\mathcal{L}_{CC} = -\frac{g}{2\sqrt{2}} W_\mu^\dagger \left[\bar{q}_u \gamma^\mu (1-\gamma_5) q_d + \bar{\nu}_l \gamma^\mu (1-\gamma_5) l \right] + \text{h.c.}$$

Quark / Lepton Universality

Left – Handed Interaction

NEUTRAL CURRENTS

$$\mathcal{L}_{\text{NC}} = -g' W_\mu^3 \bar{\psi}_1 \gamma^\mu \frac{\sigma_3}{2} \psi_1 - g' B_\mu \sum_j y_j \bar{\psi}_j \gamma^\mu \psi_j$$

Neutral Fields \rightarrow Arbitrary Combination
 (broken symmetry, since $M_Z \neq 0$)

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} \equiv \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$$

A_μ has the QED Interaction IF $g \sin \theta_W = g' \cos \theta_W = e$

$$y_1 = Q_u - \frac{1}{2} = Q_d + \frac{1}{2} \quad ; \quad y_2 = Q_u \quad ; \quad y_3 = Q_d$$

Electroweak
Unification

$$\begin{aligned} \mathcal{L}_{\text{NC}} &= -e A_\mu \sum_j \bar{\psi}_j \gamma^\mu Q_j \psi_j + \mathcal{L}_{\text{NC}}^Z \\ Q_1 &= \begin{pmatrix} Q_u & 0 \\ 0 & Q_d \end{pmatrix} ; \quad Q_2 = Q_u ; \quad Q_3 = Q_d \end{aligned}$$

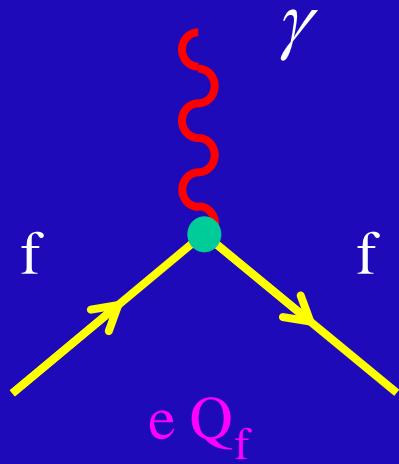
$$\begin{aligned}\mathcal{L}_{\text{NC}}^Z &= - \frac{e}{\sin \theta_W \cos \theta_W} Z_\mu \left\{ \bar{\psi}_1 \gamma^\mu \frac{\sigma_3}{2} \psi_1 - \sin^2 \theta_W \sum_j \bar{\psi}_j \gamma^\mu Q_j \psi_j \right\} \\ &= - \frac{e}{2 \sin \theta_W \cos \theta_W} Z_\mu \sum_f \bar{f} \gamma^\mu [v_f - a_f \gamma_5] f\end{aligned}$$

	q_u	q_d	ν_l	l^-
$2 v_f$	$1 - \frac{8}{3} \sin^2 \theta_W$	$-1 + \frac{4}{3} \sin^2 \theta_W$	1	$-1 + 4 \sin^2 \theta_W$
$2 a_f$	1	-1	1	-1

IF ν_R do exist: $y(\nu_R) = Q_\nu = 0 \rightarrow$ No ν_R Interactions

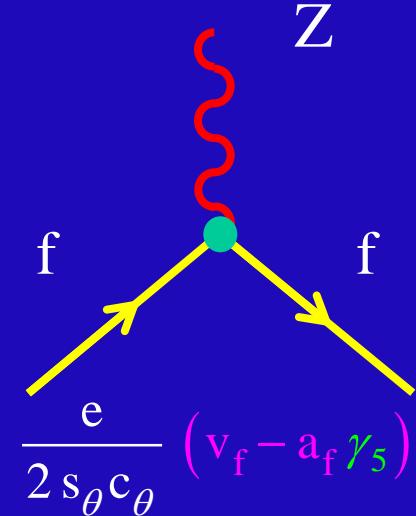
Sterile Neutrinos

NEUTRAL CURRENTS

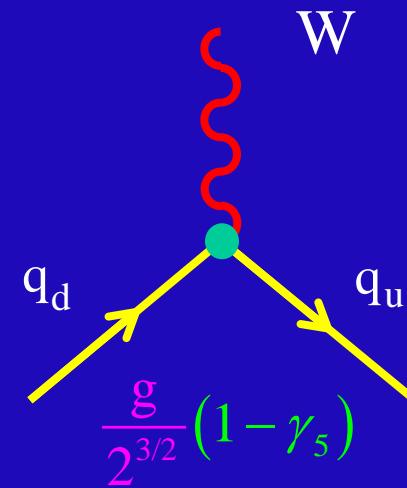
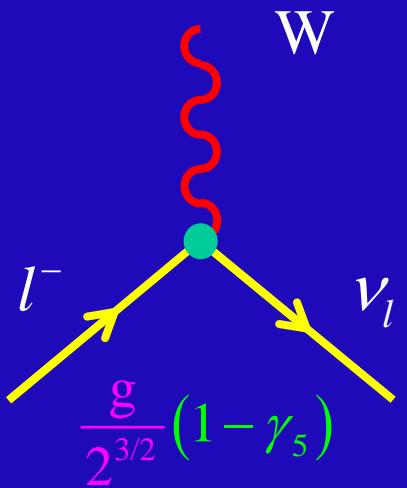


$$a_f = T_3^f = \pm \frac{1}{2}$$

$$v_f = T_3^f \left(1 - 4 |Q_f| \sin^2 \theta_w \right)$$



CHARGED CURRENTS



$y(\nu_R) = Q_{\nu} = 0 \rightarrow$ No ν_R Interactions

Sterile Neutrinos

$$\mathbf{W}_{\mu\nu} \equiv -\frac{i}{g} \left[\mathbf{D}_\mu, \mathbf{D}_\nu \right] \equiv \frac{\vec{\sigma}}{2} \cdot \vec{W}_{\mu\nu} \quad \rightarrow \quad \mathbf{U}_L \ \mathbf{W}_{\mu\nu} \ \mathbf{U}_L^\dagger \qquad ; \qquad B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu \quad \rightarrow \quad B_{\mu\nu}$$

$$W_{\mu\nu}^i = \partial_\mu W_\nu^i - \partial_\nu W_\mu^i - g \ \varepsilon^{ijk} \ W_\mu^j W_\nu^k$$

$$\boxed{\mathcal{L}_K = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{2} \text{Tr}(\mathbf{W}_{\mu\nu} \mathbf{W}^{\mu\nu}) = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} \vec{W}_{\mu\nu} \vec{W}_{\mu\nu} = \mathcal{L}_{\text{kin}} + \mathcal{L}_3 + \mathcal{L}_4}$$

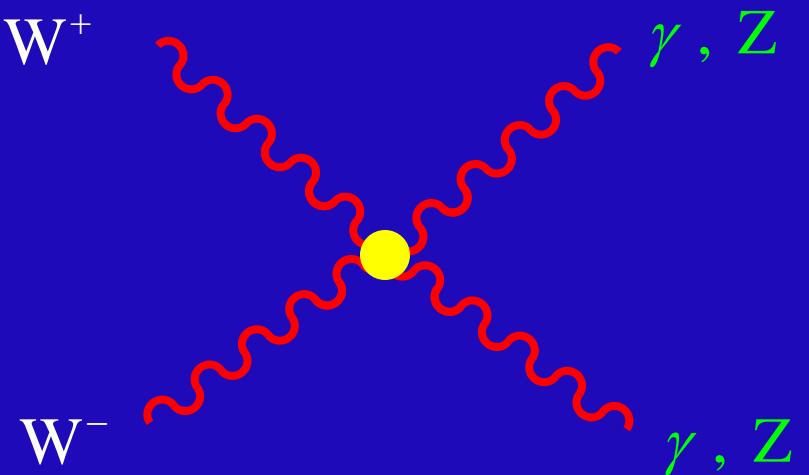
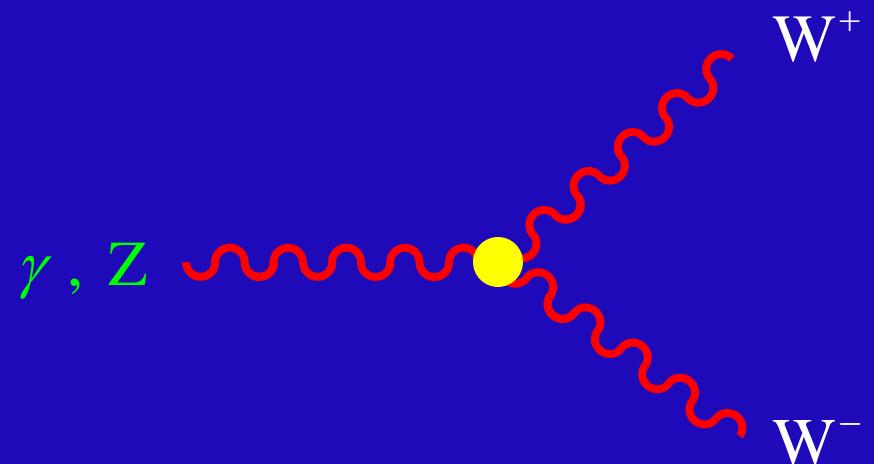
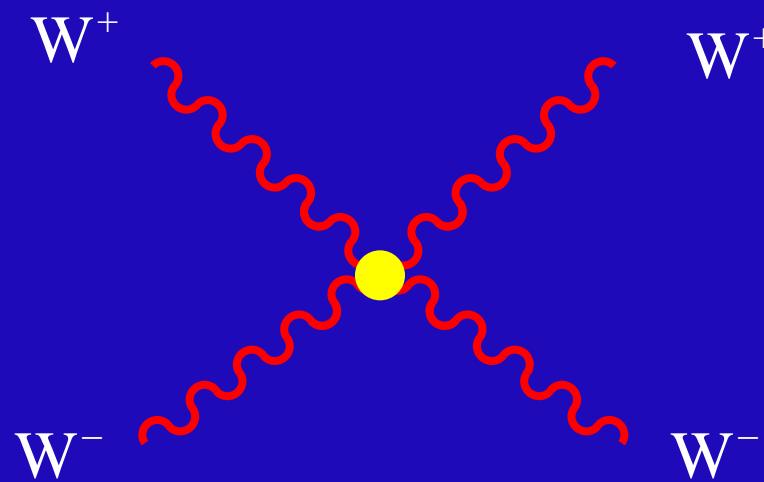
$$\mathcal{L}_3 = i e \cot \theta_W \left\{ (\partial^\mu W^\nu - \partial^\nu W^\mu) W_\mu^\dagger Z_\nu - (\partial^\mu W^{\nu\dagger} - \partial^\nu W^{\mu\dagger}) W_\mu Z_\nu + W_\mu W_\nu^\dagger (\partial^\mu Z^\nu - \partial^\nu Z^\mu) \right\}$$

$$+ i e \left\{ \left(\partial^\mu W^\nu - \partial^\nu W^\mu \right) W_\mu^\dagger A_\nu - \left(\partial^\mu W^{\nu\dagger} - \partial^\nu W^{\mu\dagger} \right) W_\mu A_\nu + W_\mu W_\nu^\dagger \left(\partial^\mu A^\nu - \partial^\nu A^\mu \right) \right\}$$

$$\mathcal{L}_4 = -\frac{e^2}{2 \sin^2 \theta_W} \left\{ \left(W_\mu^\dagger W^\mu \right)^2 - W_\mu^\dagger W^{\mu\dagger} W_\nu W^\nu \right\} - e^2 \cot^2 \theta_W \left\{ W_\mu^\dagger W^\mu Z_\nu Z^\nu - W_\mu^\dagger Z^\mu W_\nu Z^\nu \right\}$$

$$- e^2 \cot \theta_W \left\{ 2 W_\mu^\dagger W^\mu Z_\nu A^\nu - W_\mu^\dagger Z^\mu W_\nu A^\nu - W_\mu^\dagger A^\mu W_\nu Z^\nu \right\} - e^2 \left\{ W_\mu^\dagger W^\mu A_\nu A^\nu - W_\mu^\dagger A^\mu W_\nu A^\nu \right\}$$

GAUGE SELF-INTERACTIONS



PROBLEM WITH MASS SCALES

Gauge Symmetry



$$m_\gamma = 0$$

Good

$$M_W = M_Z = 0$$

Bad!



$$M_W = 80.40 \text{ GeV}$$

$$M_Z = 91.19 \text{ GeV}$$

Moreover

$$\mathcal{L}_{m_f} \equiv -m_f \bar{f} f = -m_f (\bar{f}_L f_R + \bar{f}_R f_L)$$

Also Forbidden by Gauge Symmetry



$$m_f = 0$$

$\forall f$

All Particles Massless