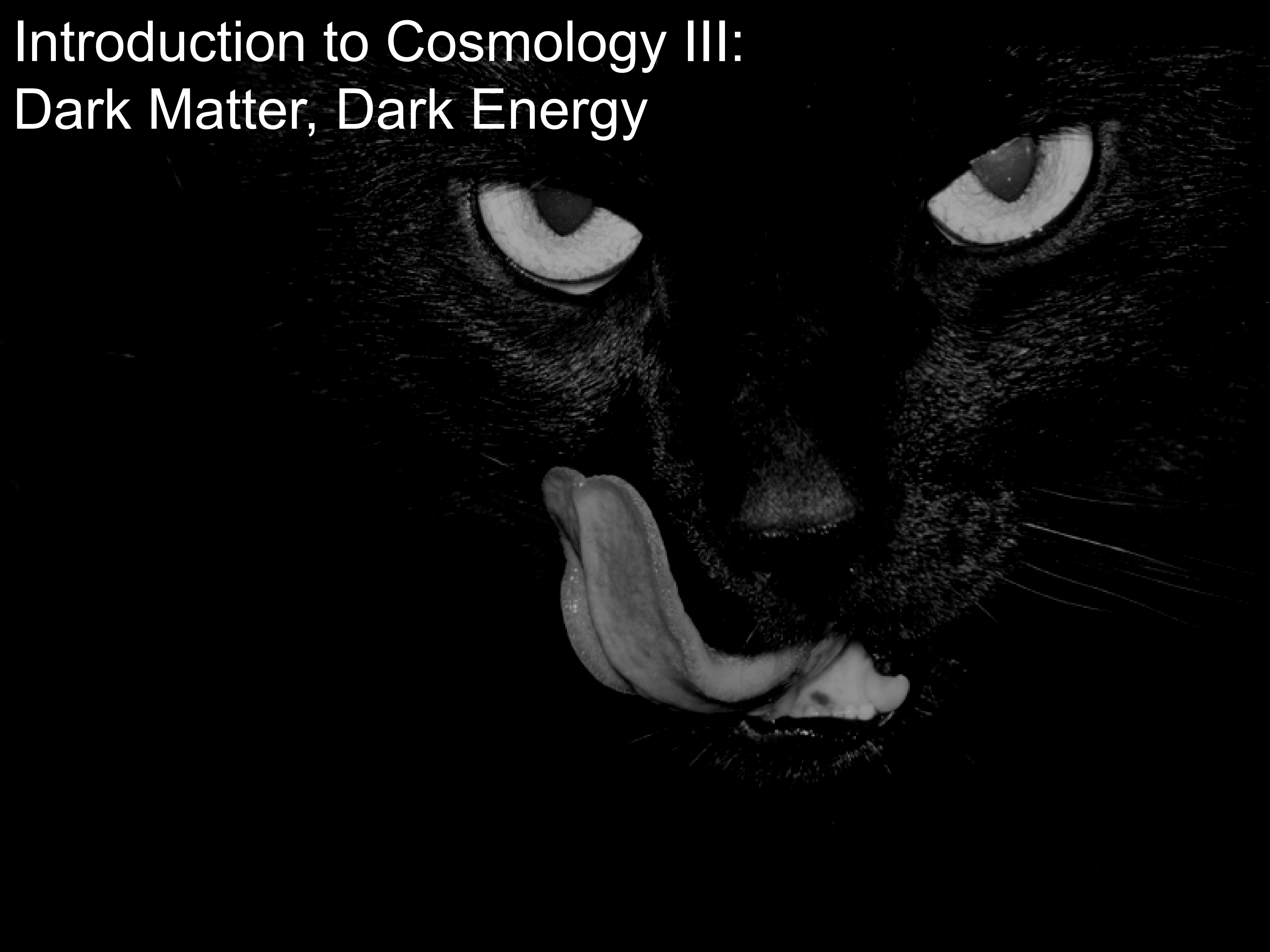
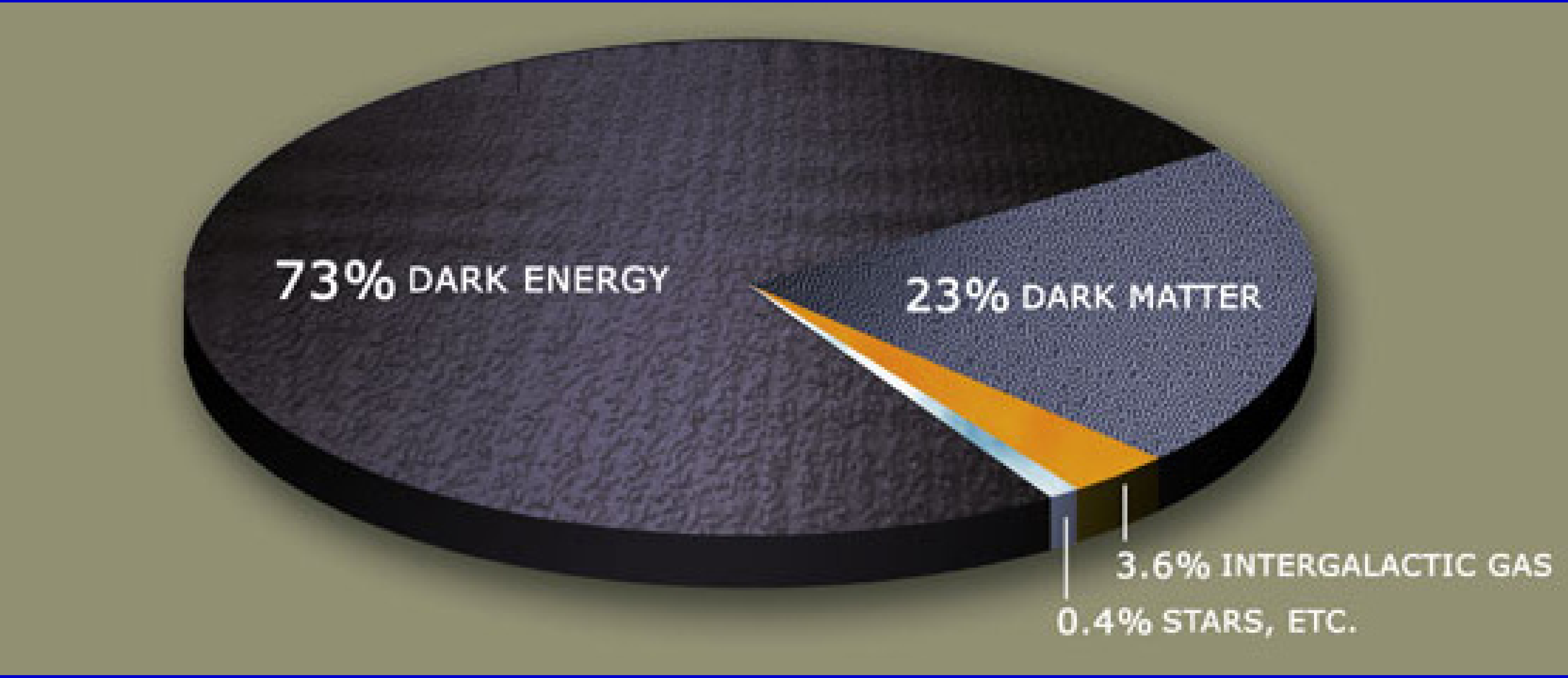
Introduction to Cosmology III: Dark Matter, Dark Energy



Λ CDM

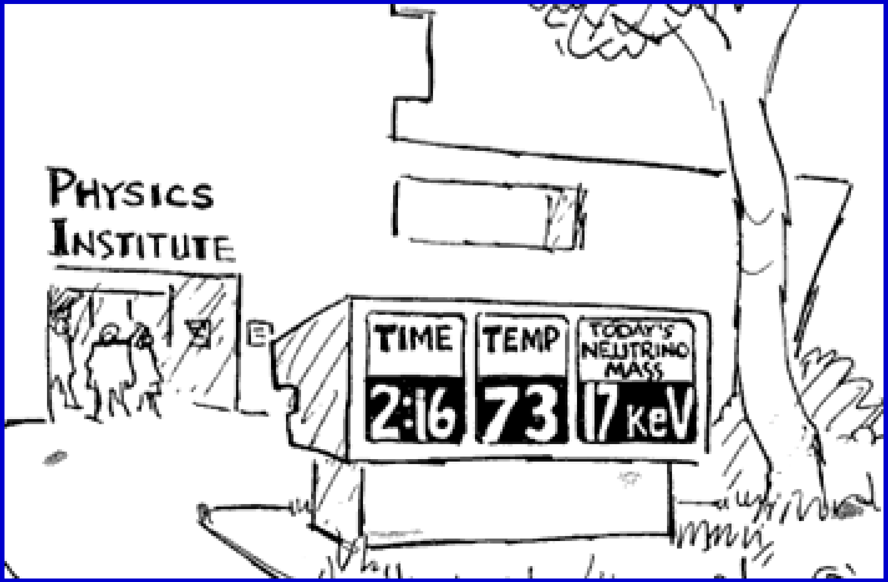
The cosmic "pie chart":



(The "pie slice" for photons is too small too be seen.)

Leading candidates for **dark matter**:
particles that interact only through the weak force.

For instance:
massive neutrinos.



During the 1st second after the Big Bang,
the universe was opaque to neutrinos.

There should exist a Cosmic **Neutrino** Background,
analogous to the Cosmic Microwave Background.

The number density of neutrinos in the Cosmic Neutrino Background can be calculated:

$$n_{C\nu B} = 3\left(\frac{3}{11}\right)n_{CMB} = 3.36\times10^{8}\,m^{-3}$$

If the average neutrino mass is $m_\nu c^2 \approx 4$ eV, then neutrinos provide $\Omega_{dm} \approx 0.23$.

However, neutrinos can't provide all the dark matter.

Neutrinos with $m_\nu c^2 \leq 4$ eV constitute **Hot Dark Matter**; astronomical evidence shows that the dark matter is mostly **Cold Dark Matter (CDM)**.

Hot dark matter consists of particles that were **relativistic** when they decoupled from other particles (t~1 sec), but have now cooled to be non-relativistic.

Cold dark matter was already **non-relativistic** at the time of decoupling.

Why do we care about hot vs. cold?

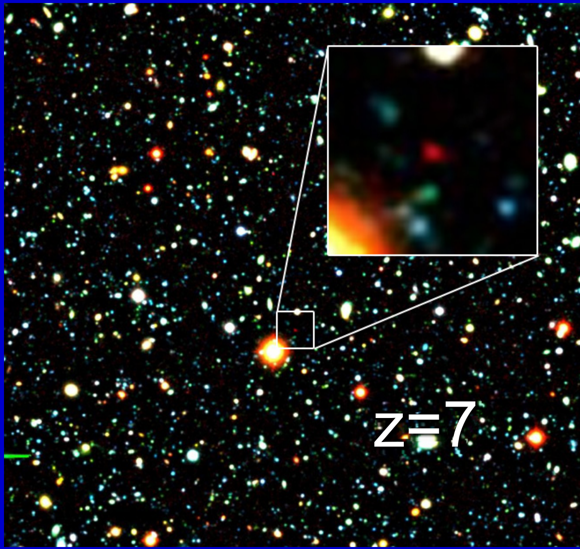
Free streaming of hot dark matter during its relativistic phase smears out short-wavelength density perturbations; anything smaller than today's superclusters are wiped out ($m_\nu c^2 \sim 4$ eV).

In a universe where the dark matter is **hot**, superclusters form first; they then fragment to form clusters, then individual galaxies.

Not observed.

In a universe where the dark matter is **cold**, galaxies form first; they then aggregate into clusters, which then form superclusters.

This **is** observed:
galaxies are seen at large redshift,
and clusters at intermediate redshift;
superclusters are just now collapsing.



The leading candidates for **Cold** Dark Matter are the class of particles known to astronomers as WIMPs (**W**eakly **I**nteracting **M**assive **P**articles).

Supersymmetry predicts various massive non-baryonic particles such as neutralinos. WIMPs might be the lightest neutralino.

“Dark matter is elusive”: Direct detection experiments for WIMPs are inconclusive so far.

Dark energy can be defined as a component of the universe that causes its expansion to speed up ($\ddot{a} > 0$).

The Friedman equation is 1 equation in 2 unknowns:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon(t) - \frac{\kappa c}{R_0^2 a(t)^2}$$

We need another equation, such as the **fluid equation**:

Adiabatic expansion:
no heat flow

$$\dot{Q} = \dot{E} + P\dot{V}$$

$$\text{or} \quad \dot{\varepsilon} = -3\frac{\dot{a}}{a}(\varepsilon + P), \quad \text{where} \quad \varepsilon = E/V$$

If you prefer, the Friedman equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon - \frac{\kappa c}{R_0^2 a^2}$$

and the fluid equation

$$\dot{\varepsilon} = -3\frac{\dot{a}}{a}\left(\varepsilon + P\right)$$

can be combined to form the acceleration equation:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\varepsilon + 3P)$$

(Exercise left for the reader!)

Using the acceleration equation,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\varepsilon + 3P)$$

we can rewrite our definition of dark energy:

Dark energy is something with $P < -\varepsilon/3$.

How does this tie in to Einstein's cosmological constant, Λ?

What Einstein thought (ca. 1915):

The universe contains both light and matter.

There is much more matter than light.

The universe is neither expanding nor contracting.

Einstein's difficulty:

a universe dominated by non-relativistic, low-pressure matter must be expanding or contracting.

Einstein's solution was to add a fudge factor, called Λ, or the "cosmological constant", to his field equation.

The added factor of Λ changes the Friedman equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon - \frac{\kappa c}{R_0^2 a^2} + \frac{\Lambda}{3}$$

The fluid equation is unchanged by the addition of Λ; the acceleration equation becomes:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\varepsilon + 3P) + \frac{\Lambda}{3}$$

How can Λ produce a static universe?

Consider a universe containing pressureless matter and a cosmological constant. Its acceleration equation is:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho + \frac{\Lambda}{3}$$

The acceleration $\ddot{a}$ vanishes if we set $\Lambda = 4\pi G\rho$.

For a static universe, we want $\dot{a}$ to vanish, as well as $\ddot{a}$. Let's go back to the Friedman equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon - \frac{\kappa c}{R_0^2 a^2} + \frac{\Lambda}{3}$$

In a static universe, containing only matter and Λ,

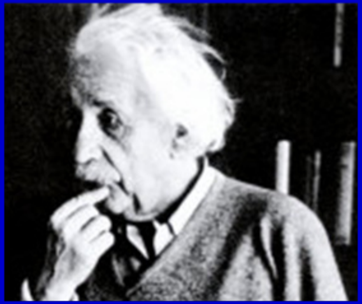
$$0 = \frac{8\pi G}{3}\rho - \frac{\kappa c^2}{R_0^2} + \frac{4\pi G}{3}\rho$$

Thus, Einstein's static universe must be positively curved (κ=+1), with a radius of curvature

$$R_0 = \frac{c}{(4\pi G\rho)^{1/2}} = \frac{c}{\Lambda^{1/2}}$$

Although Einstein published his static, positively-curved universe in 1917, he didn't like it.

Why was Einstein not happy?



1) He thought that Λ was “gravely detrimental to the formal beauty of the theory”.

2) His static universe is in **unstable** equilibrium.

When Hubble published Hubble’s Law in 1929, Einstein discarded Λ, which he called “the greatest blunder of my career”.

The cosmological constant is back in fashion, but today's cosmologists think of it differently. Consider the Friedman equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\varepsilon - \frac{\kappa c}{R_0^2 a^2} + \frac{\Lambda}{3}$$

and the acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\varepsilon + 3P) + \frac{\Lambda}{3}$$

The cosmological constant has the same effect as a component with energy density

$$\varepsilon_\Lambda = \frac{\Lambda c^2}{8\pi G} = \text{constant}$$

and pressure $P_\Lambda = -\frac{\Lambda c^2}{8\pi G} = -\varepsilon_\Lambda = \text{constant}$

When we think of Λ as yet another physical component of the universe, instead of a disreputable fudge factor, we see that it's a form of **dark energy**.

$P_\Lambda = -\varepsilon_\Lambda$ fills our definition of dark energy ($P < -\varepsilon/3$).

In a flat universe containing only Λ,

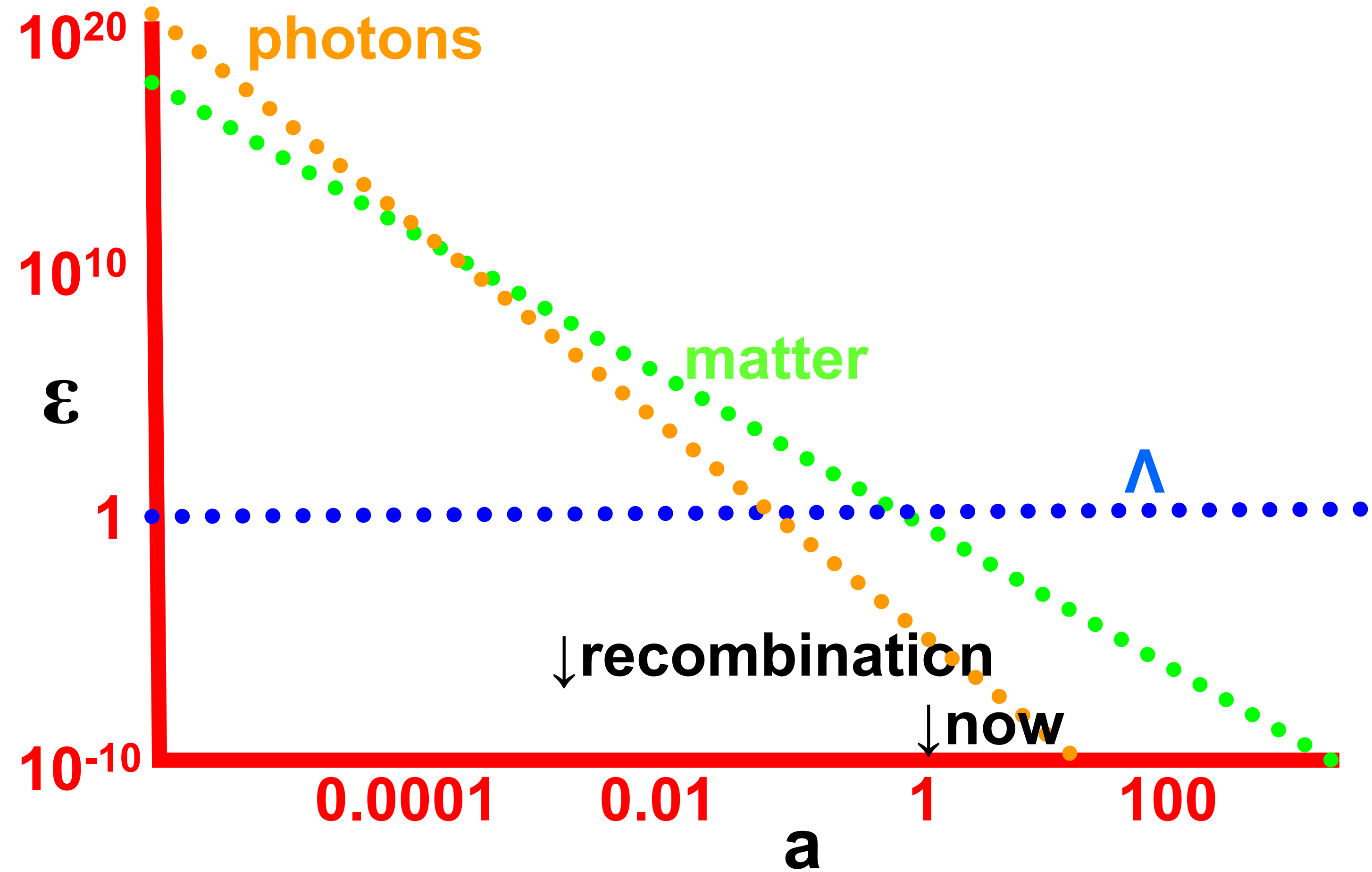
$$\frac{\dot{a}}{a} = \left(\frac{8\pi G}{3c^2} \varepsilon_\Lambda \right)^{1/2} = \text{constant}$$

Exponential growth!

$$\varepsilon_{photons} \propto a(t)^{-4}$$

$$\varepsilon_{matter} \propto a(t)^{-3}$$

$$\varepsilon_{\Lambda} = constant$$



What is this miracle substance that I'm calling the cosmological constant?

Its density remains constant as the universe expands.

To keep at constant density, it has to have $P_\Lambda = -\varepsilon_\Lambda$.

$$\dot{\varepsilon} = -3\,\frac{\dot{a}}{a}\left(\varepsilon + P\right)$$

This is some really weird stuff, by everyday standards.

Some cosmologists equate Λ with the **vacuum energy** of the universe.

The vacuum is seething with virtual particle – antiparticle pairs. The energy density ε_{vac} of these virtual particles is a quantum phenomenon.

The bad news: the only natural values for ε_{vac} are zero and the Planck density, $E_{Pl}/l_{Pl}^{3} \sim 3 \quad 10^{123}$ GeV m^{-3}.

The measured density of Λ is

$$\varepsilon_{\Lambda} = 0.73\ \varepsilon_{c,0} = 3.9 \text{ GeV m}^{-3}.$$

How do we know whether the cosmological constant actually exists?

Space is nearly flat, thus $\Omega \approx 1$.

However, all the matter and radiation in the universe adds up to $\Omega \lesssim 0.3$.

Something must provide the rest of the energy density; maybe it's a cosmological constant.

This is a weak argument.

Let's look for stronger arguments.

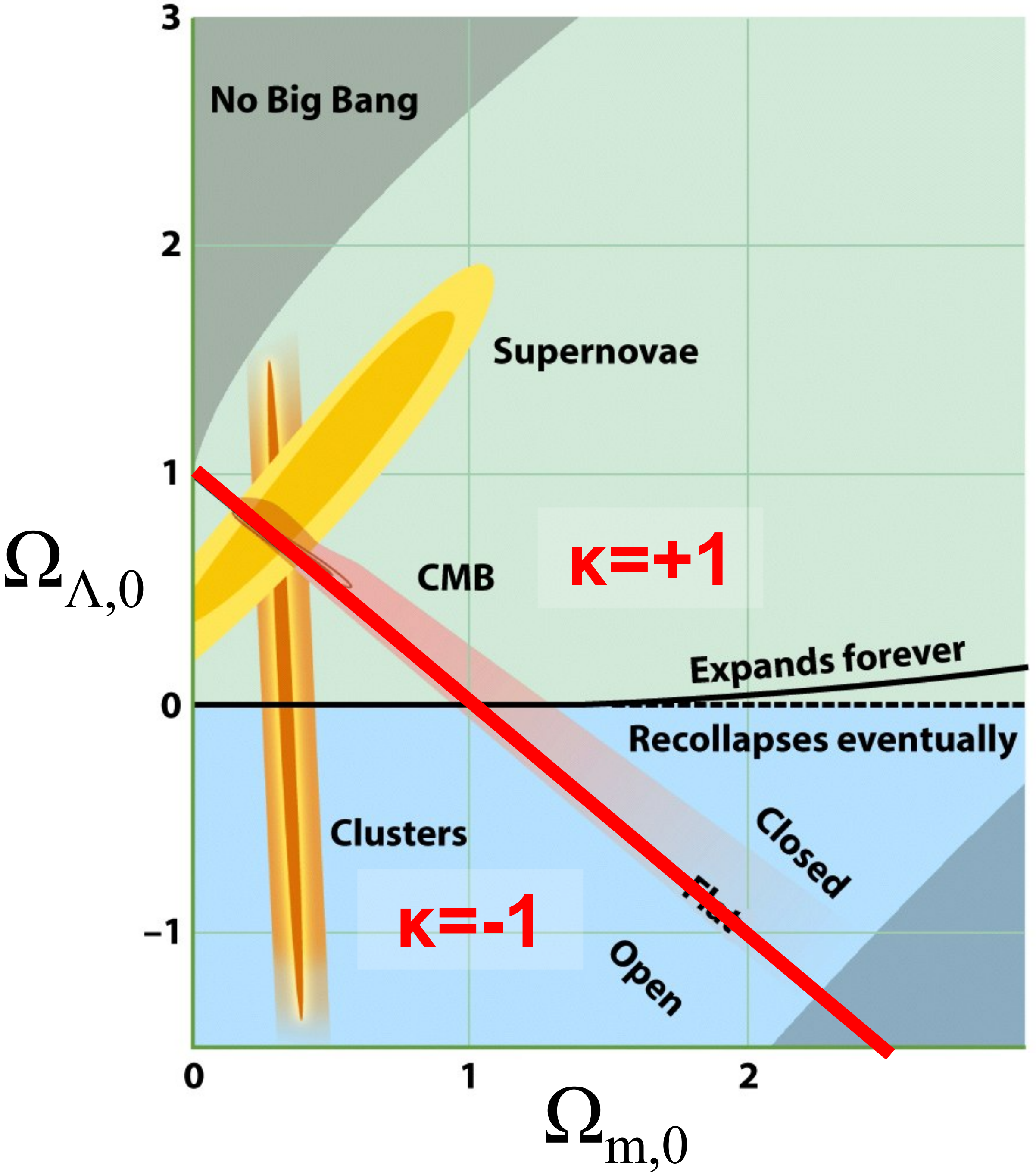
Consider a universe containing **matter**, with density parameter $\Omega_{m,0}$, and Λ, with density parameter $\Omega_{\Lambda,0}$.

Its Friedman equation can be written in the form

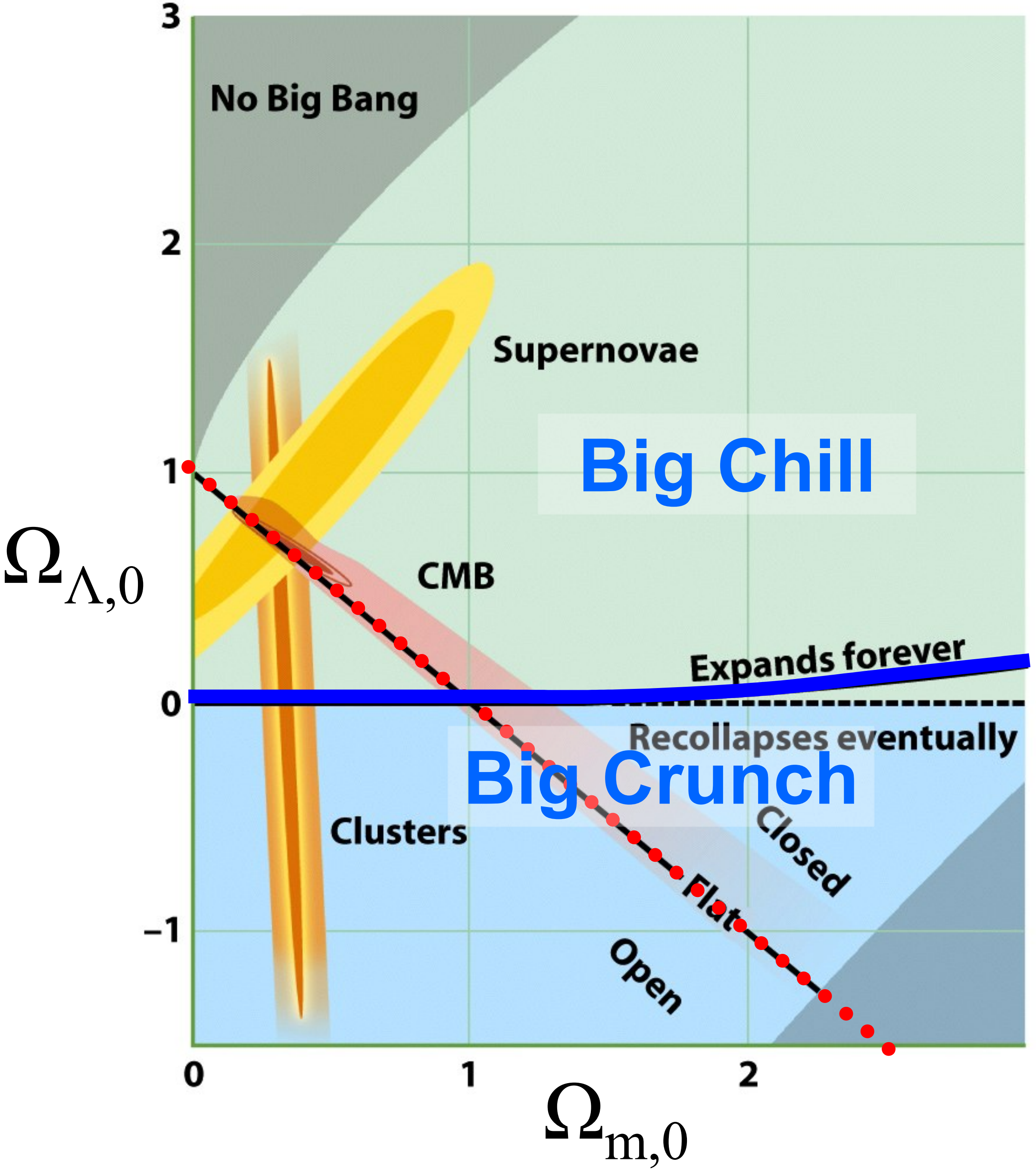
$$\frac{H^2}{H_0^2} = \frac{\Omega_{m,0}}{a^3} + \frac{1 - \Omega_{m,0} - \Omega_{\Lambda,0}}{a^2} + \Omega_{\Lambda,0}$$

mass term (+) | curvature term (+ or -) | Λ term (+ or -)

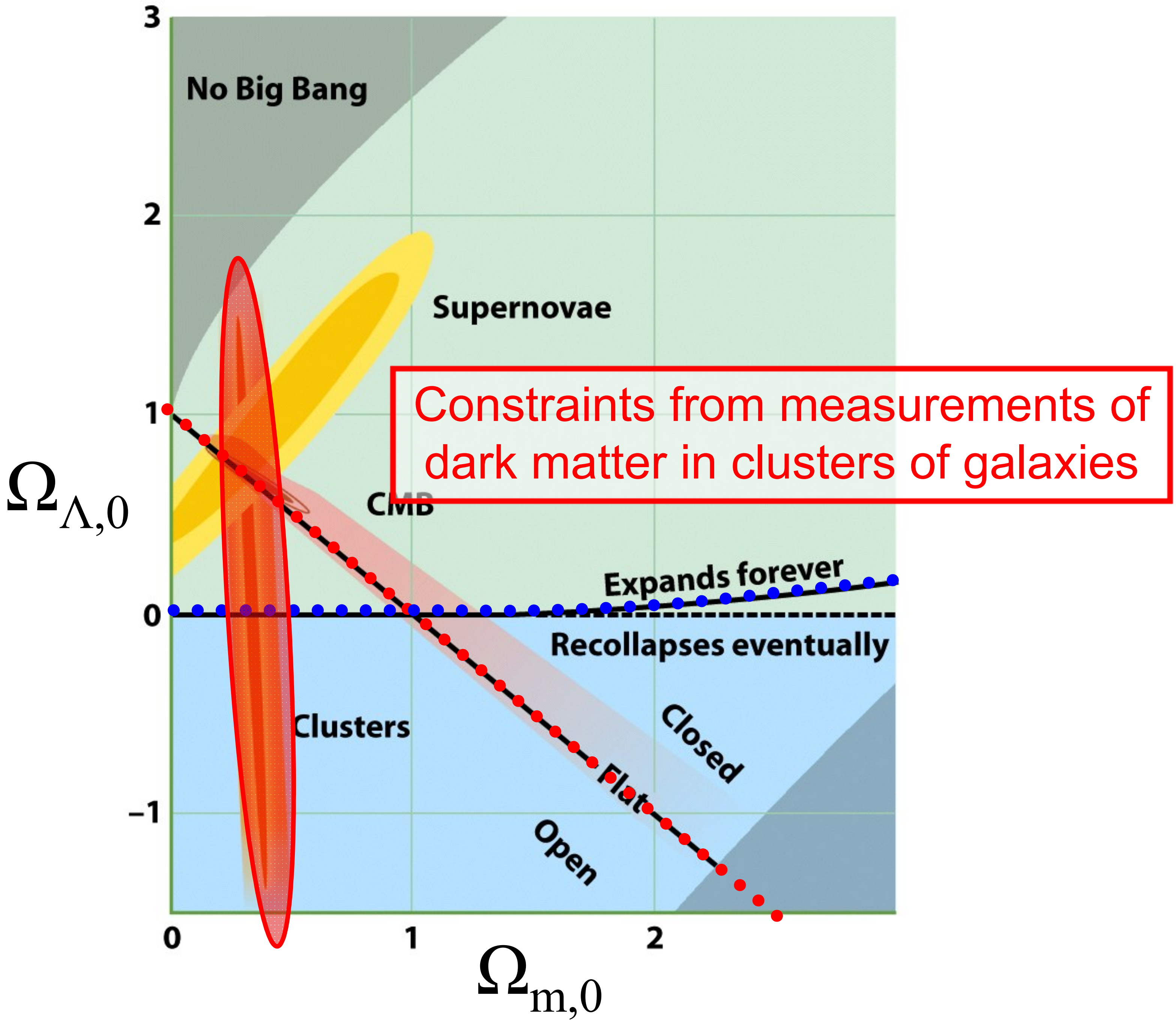
Curvature is **not** destiny: Positively curved universes can expand forever or recollapse, for example.



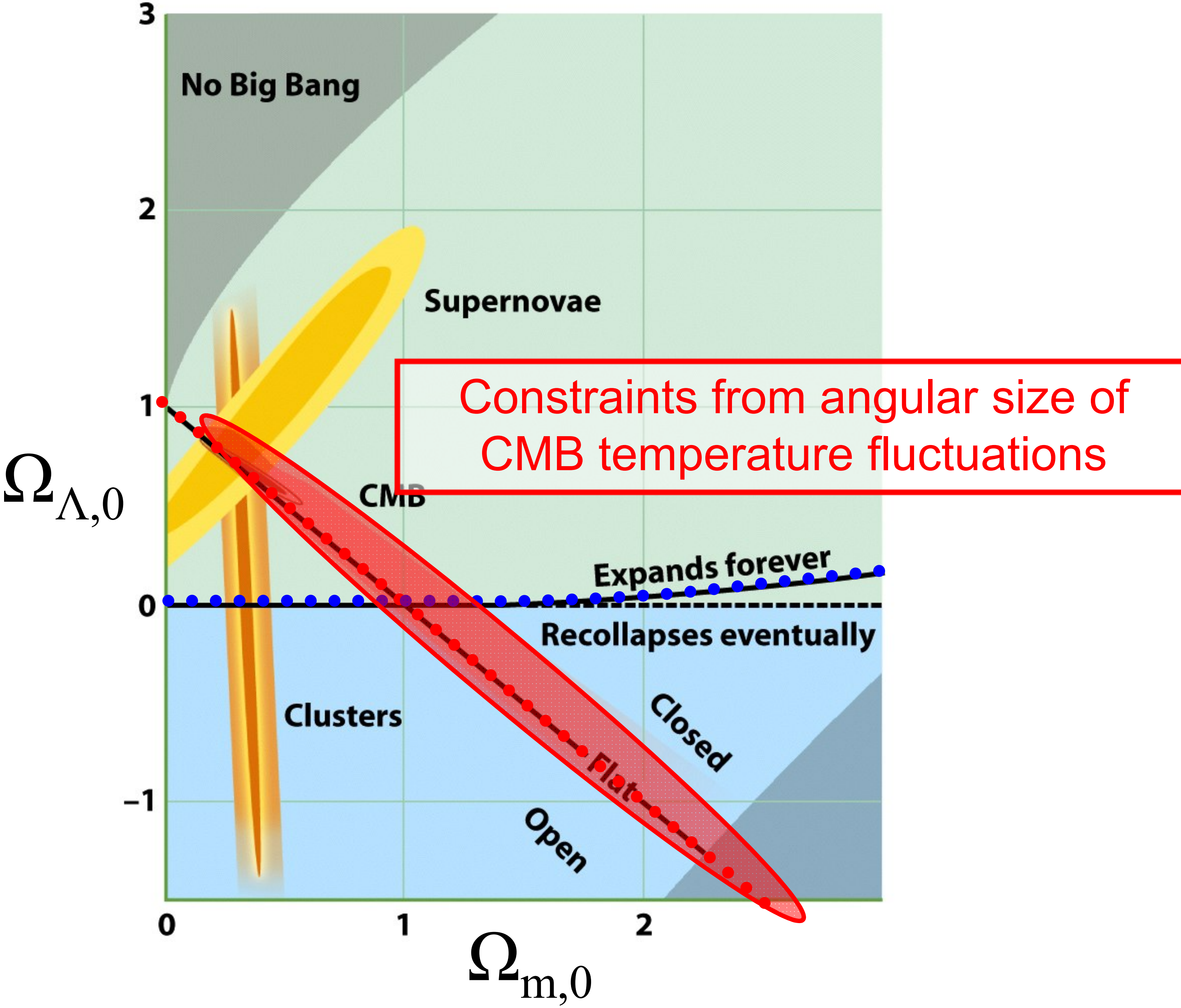
3
2
1
0
−1
No Big Bang
Supernovae
CMB
κ=+1
Expands forever
Recollapses eventually
Clusters
Closed
Flat
Open
κ=-1
0
1
2
$\Omega_{\Lambda,0}$
$\Omega_{m,0}$



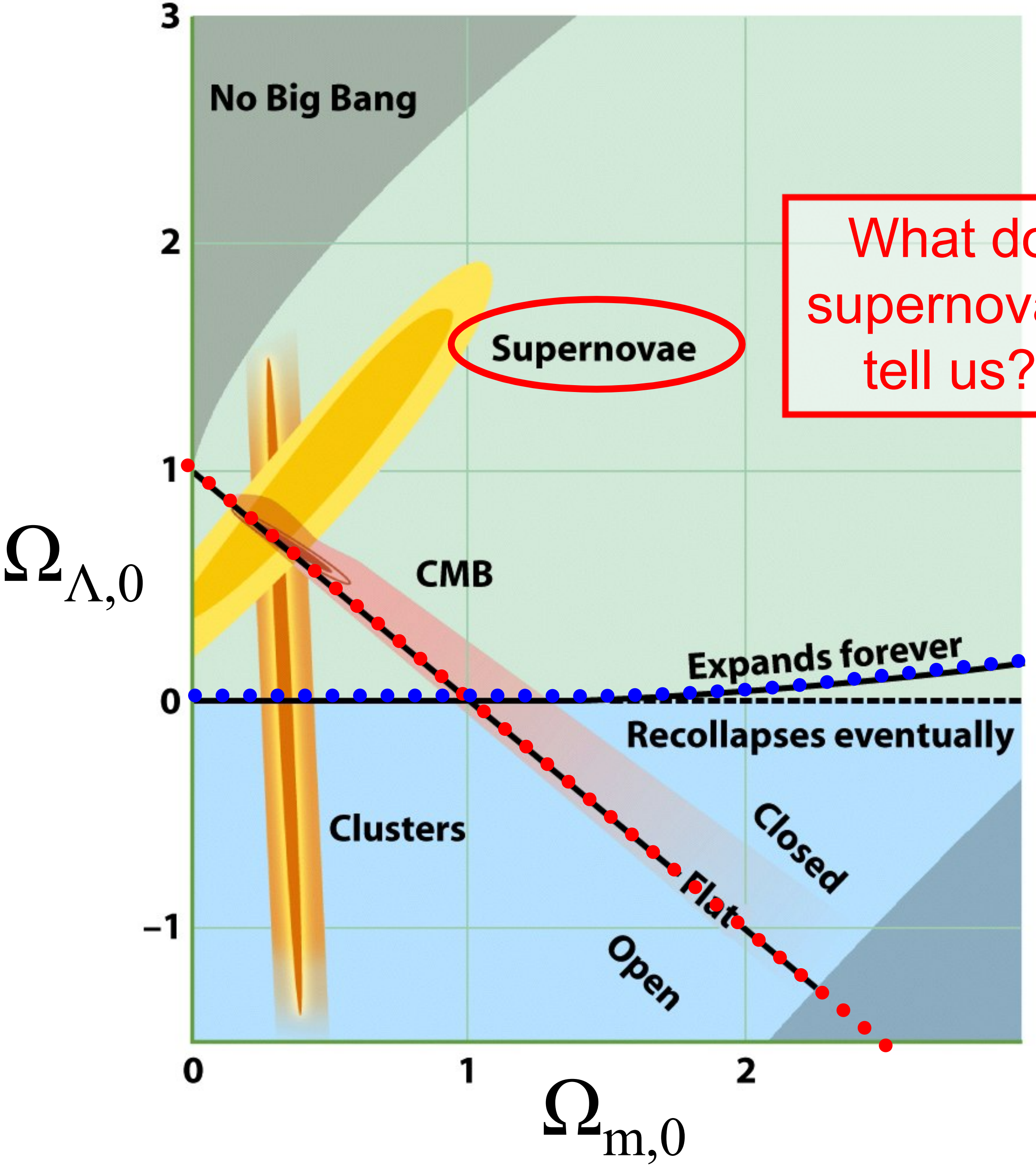
No Big Bang
Supernovae
Big Chill
CMB
Expands forever
Recollapses eventually
Big Crunch
Clusters
Closed
Flat
Open
3
2
1
0
–1
0
1
2
$\Omega_{\Lambda,0}$
$\Omega_{m,0}$



3
2
1
0
−1
No Big Bang
Supernovae
Constraints from measurements of dark matter in clusters of galaxies
CMB
Expands forever
Recollapses eventually
Clusters
Closed
Flat
Open
0
1
2
$\Omega_{\Lambda,0}$
$\Omega_{m,0}$



Constraints from angular size of CMB temperature fluctuations
No Big Bang
Supernovae
CMB
Expands forever
Recollapses eventually
Clusters
Closed
Flat
Open
3
2
1
0
–1
0
1
2
$\Omega_{\Lambda,0}$
$\Omega_{m,0}$



What do supernovae tell us?

Type Ia supernovae are white dwarfs that have gone over the Chandrasekhar limit; this triggers a runaway fusion reaction.

Type Ia supernovae are excellent “standard candles”; their luminosity is large (~4 $\times 10^9$ L_{sun}) and standardized.

You know the absolute magnitude M of a supernova. You see one go off in a distant galaxy with redshift z; you measure its apparent magnitude m.

In a **static** Euclidean universe, the relation between absolute magnitude and apparent magnitude is

$$m - M = 5 \log_{10}\left(\frac{d}{1\,\mathrm{Mpc}}\right) - 25$$

where d is the "proper distance" (the distance you would measure if you could stop the expansion of the universe and stretch a tape measure to the supernova).

In an **expanding** Euclidean universe, part of the dimming of a supernova is due to the redshift z, so

$$m - M = 5 \log_{10}\left(\frac{d(1+z)}{1\,\mathrm{Mpc}}\right) - 25$$

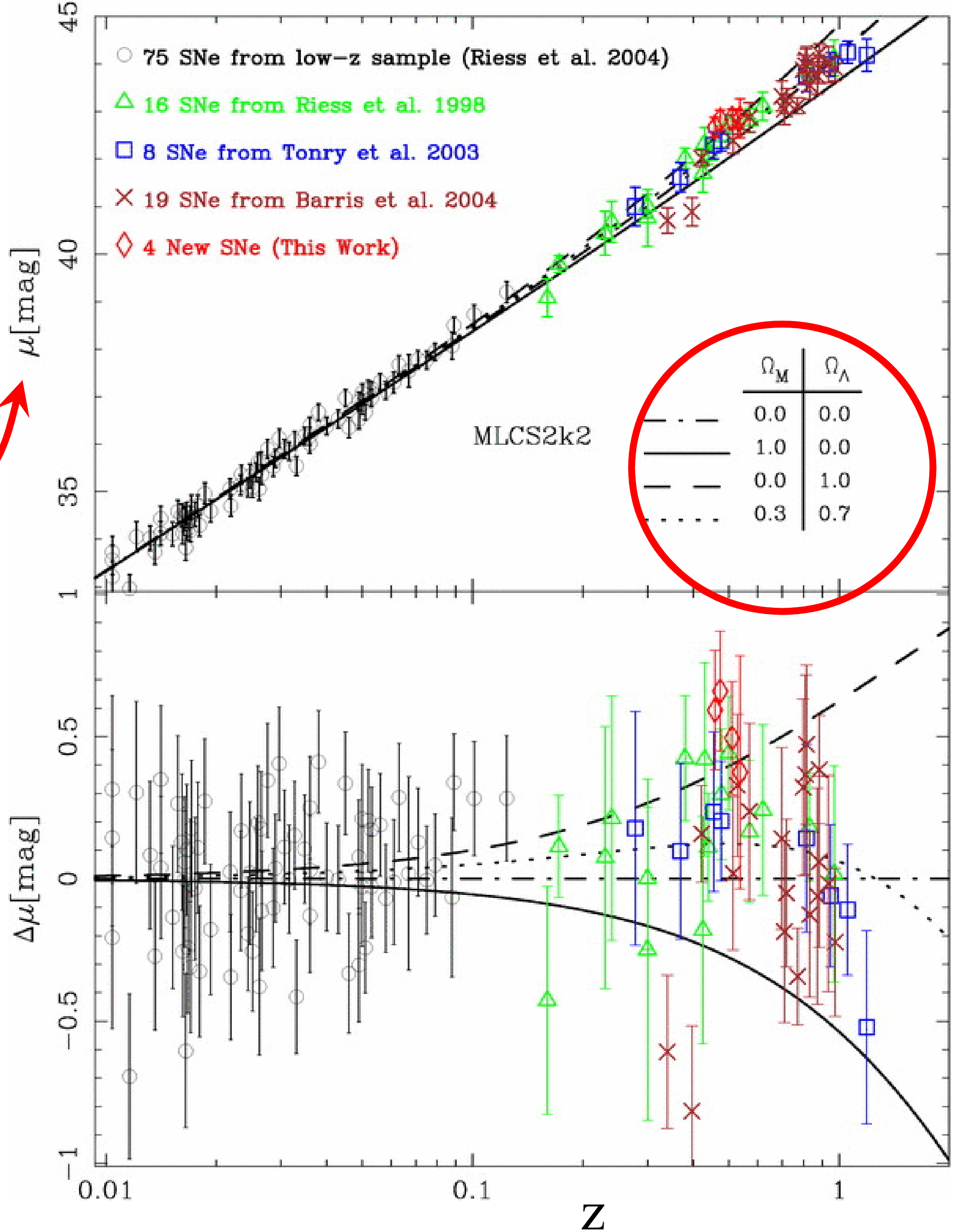
In the limit $z \ll 1$, $d \approx (c/H_0)z$, so a plot of m versus $\log z$ should be linear for small redshift:

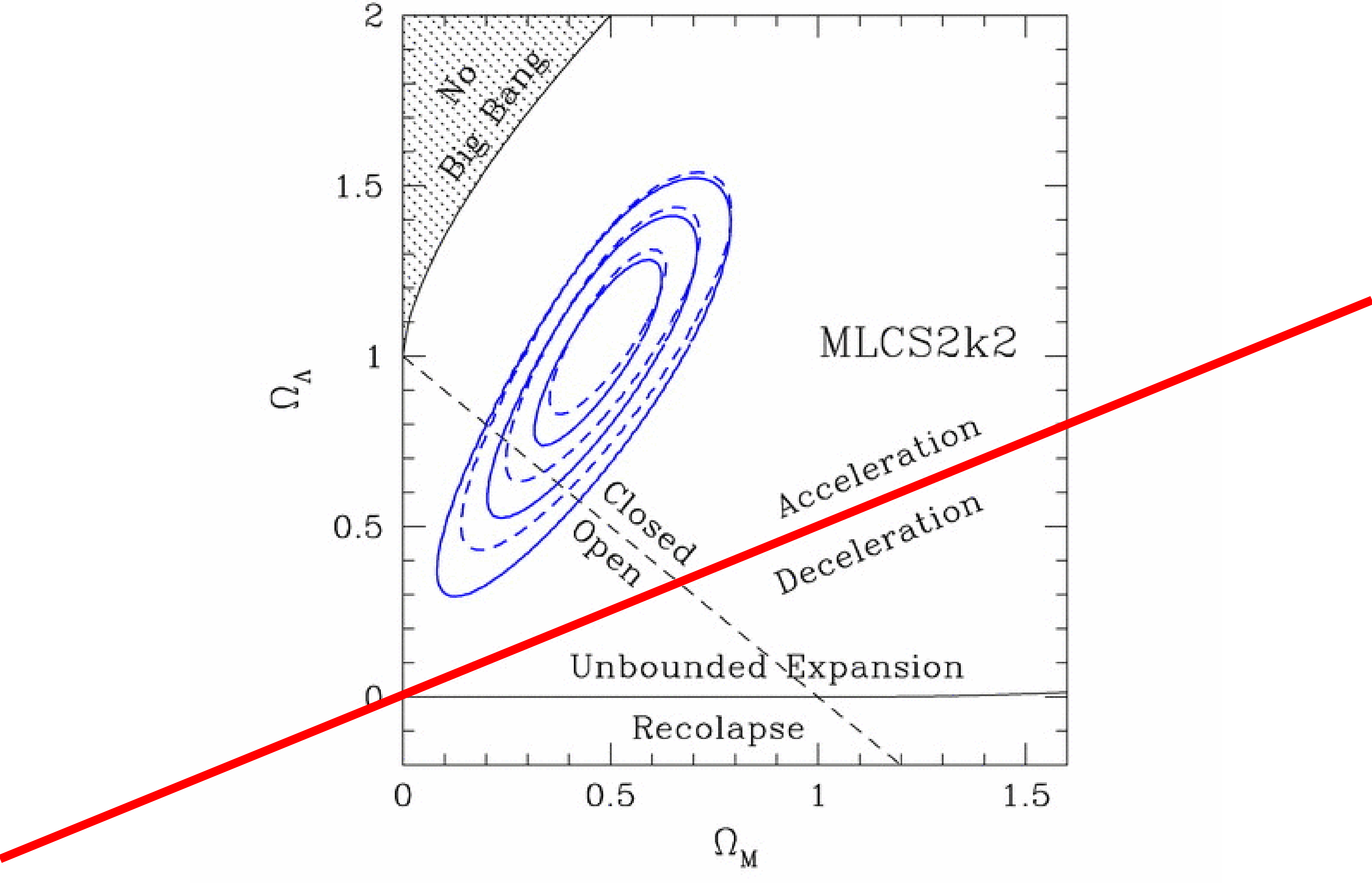
$$m - M \approx 5\log z + 5\log\left(\frac{c}{H_0}\right) - 25$$

However, in the limit of large z, the relation between proper distance and redshift goes nonlinear, and so does the relation between m and $\log z$.

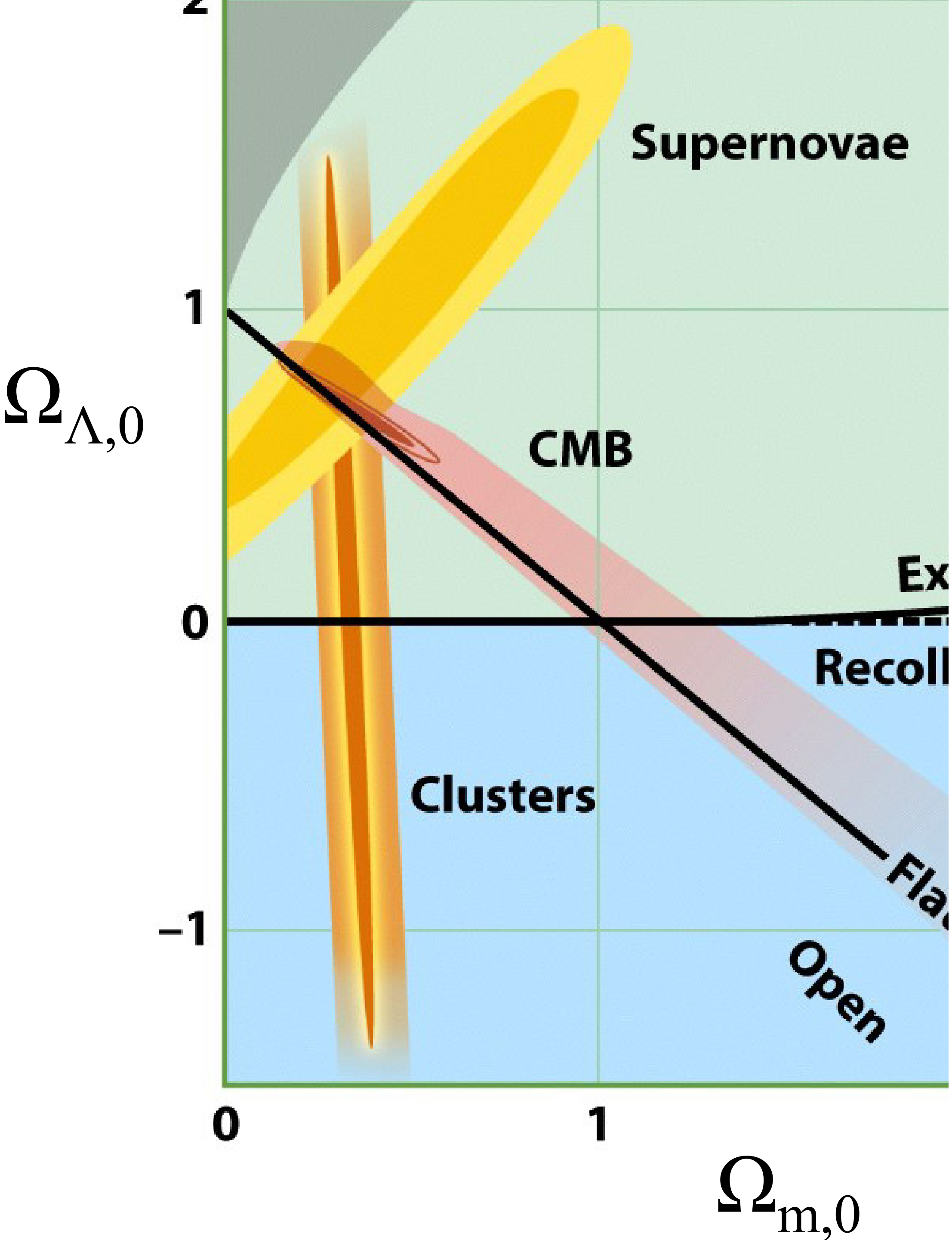
Results from recent supernova surveys:

μ = m-M





No Big Bang
MLCS2k2
Acceleration
Deceleration
Closed
Open
Unbounded Expansion
Recolapse
Ω_Λ
Ω_M
2
1.5
1
0.5
0
0
0.5
1
1.5



2
1
0
−1
Supernovae
CMB
Clusters
Ex
Recoll
Fla
Open
0
1
$\Omega_{\Lambda,0}$
$\Omega_{m,0}$

I thank you…

…and the Newtonian bear thanks you.

The inevitable question:

"Where does all the energy come from when a Λ-dominated universe expands?"

The flippant answer:

"The same place the energy goes to when a radiation-dominated universe expands."

The more serious answer:

"Energy is not a globally conserved property in GR."

Suppose I have a mirrored box full of photons.

If I increase the volume of the box, the equation

$$0 = \dot{E} + P\dot{V}$$

tells me that the energy lost by the photons is doing PdV work on the walls of the box.

Suppose I have a universe full of photons.

If I increase the volume of the universe, the equation

$$0 = \dot{E} + P\dot{V}$$

can still be used (it’s a valid equation in GR), but the photons aren’t doing PdV work. The universe has no walls.